1. Distribution of the Smallest Geometric. Let $X_i \sim Ge(p_i)$ for $i = 1, \ldots, n$. Here $Ge(p_i)$ is geometric distribution defined by

$$P(X_i = k) = q_i^{k-1}p_i, \quad k = 1, 2, 3, \ldots \quad p_i + q_i = 1,$$

and represents the number of trials needed to see the first success.

Let $X_{(1)}$ be the minimum of $X_1, \ldots, X_n$.

Show $X_{(1)} \sim Ge(1 - q_1 q_2 \ldots q_n)$

**Hint:** (i) Represent $P(X_{(1)} = k) = P(X_{(1)} \geq k) - P(X_{(1)} \geq k + 1)$. (ii) Argue $P(X_{(1)} \geq m) = P(X_1 \geq m) P(X_2 \geq m) \ldots P(X_n \geq m)$. (iii) Show $P(X_i \geq m) = q_i^{m-1}$.

2. Batteries. A machine uses 12 batteries. Because it has so many, it does not shut off until half of the batteries are dead.

All batteries are independent and have uniformly distributed lifetimes on the interval $[.5, 1]$ years.

(a) Find the probability that the machine shuts before 0.75 years of work?

(b) What is the expected time when the sixth battery dies?

Hint: $T_1, \ldots, T_{12} \sim f$, where $f = 2$, if $0.5 \leq t \leq 1$, and $F(t) = 2t - 1$, if $0.5 \leq t \leq 1$. Find distribution of $T_{(6)}$.

3. Limiting Law for Median of a $t_3$-Sample. Let $X_1, X_2, \ldots, X_n$ be a sample from $t$ distribution with 3 degrees of freedom. The density of this distribution is

$$f(x) = \frac{2}{\pi \sqrt{3} \left(1 + \frac{x^2}{3}\right)^2}, \quad -\infty < x < \infty.$$

(a) Find asymptotic distribution for $\sqrt{n}M_n$, where $M_n$ is the sample median, $X_{[n/2]:n}$.

(b) Using result in (a), approximate the probability

$$\mathbb{P}(-0.2 < M_{100} < 0.2).$$